

# A NEW GLOBAL TIME DOMAIN ELECTROMAGNETIC SIMULATOR OF MICROWAVE CIRCUITS INCLUDING LUMPED ELEMENTS BASED ON FINITE ELEMENT METHOD.

K.Guillouard\*, M.F. Wong\*, Member IEEE, V. Fouad Hanna\*, Fellow IEEE,  
J. Citerne\*\*

\*France Télécom - CNET, Paris, France

\*\*INSA, CNRS, URA 834, Rennes, France

## ABSTRACT

This paper proposes an extension of the Finite Element Time Domain (FETD) method for the global electromagnetic (EM) analysis of complex inhomogeneous microwave distributed circuits, containing linear or non linear lumped elements. This technique combines Maxwell's equations and circuit equations, using directly SPICE software. Results are given for a capacitor, a resistor as well as a Schottky diode.

applied in the frequency domain [2]. Moreover, it is shown that the FETD-SPICE combination can be thought of, as a generalization of the FDTD-SPICE combination [3]. Therefore, the proposed method presents the advantage to be general as it allows the characterization of complex arbitrary shaped structures, including any kind of lumped element, linear or non linear, passive or active, without needing additional software development, especially to model non linear lumped part. To prove the efficiency of this new method, examples are also provided.

## INTRODUCTION

Time domain numerical methods based on Maxwell's equations have been widely used to solve transient or large band frequency EM problems. Recently, efforts have been devoted to adapt these fullwave analyses to the characterization of complex and highly integrated microwave devices, including distributed as well as lumped circuits. Among these solutions, the FDTD extension is the most commonly proposed [4-6]. But to our knowledge, few investigations have been published concerning such an extension using the FETD [1].

So in this paper, a new time domain technique combining the FETD method and SPICE software is presented. The FETD uses mixed finite elements, namely edge elements and face elements to represent the electric field and the magnetic induction respectively. In fact, our technique extends, in the time domain, the concept of the coupling between Maxwell's equations and circuit equations, already described and successfully

## OUTLINE OF THE TECHNIQUE

### 1. Finite element method in the time domain

In the time domain, the EM problem is described with the curl Maxwell-Faraday (M-F) (1a) and the curl Maxwell-Ampere (M-A) (1b) equations :

$$\text{curl} \vec{E} = -\partial_t \vec{B} \quad (1a)$$

$$\text{curl} \frac{\vec{B}}{\mu} = \partial_t \epsilon \vec{E} + \vec{J} \quad (1b)$$

Edge elements and face elements have been shown to be adequate basis vectorial fields for the Finite Element Method that can be applied to these equations [1]. The M-F and M-A equations can be put in matrix form respectively as :

$$\vec{C} \vec{e} = -\partial_t \vec{b} \quad (2a)$$

$$\vec{C}^t \vec{\mu}^{-1} \vec{b} = \partial_t \vec{\epsilon} \vec{e} + \vec{i} \quad (2b)$$

where the vectors of unknowns,  $\bar{e}$  and  $\bar{b}$ , are the circulations of the electric field along the edges (in Volts) and the fluxes of the magnetic induction across the facets (in Webers) of the mesh respectively. The vector  $\bar{i}$  (in Amperes) is the conduction current vector due to problem boundary conditions, like excitation current sources. These two equation systems lead to the FETD algorithm adopting a leap-frog scheme in the time domain [1].

## 2. SPICE-FETD combination

A lumped element is a circuit element, whose dimensions are very small compared to the wavelength. Its current/voltage characteristic is generally expressed as :

$$i_c = f(u_c) \quad (3)$$

Thus, to account for the effect of the insertion of such a component into a distributed circuit part, an edge  $e$  is compared to a port : the associated conduction current, appearing in the M-A system (2b), is modified by adding the current flowing in the lumped element :

$$i'_e = i_e + i_c = \left( \bar{C}^t \bar{\mu}^{-1} \bar{b} \right)_e - \partial_t (\bar{\epsilon} \bar{e})_e + i_c \quad (4)$$

The difficulty lies in the implementation of the modified equation (4) in the leap-frog scheme in the time domain, due to the inversion of the mass matrix  $\bar{\epsilon}$ . The solution consists in splitting the M-A system (2b) as :

$$\begin{bmatrix} \left( \bar{C}^t \bar{\mu}^{-1} \bar{b} \right)_i \\ \left( \bar{C}^t \bar{\mu}^{-1} \bar{b} \right)_c \end{bmatrix} = \partial_t \begin{bmatrix} \bar{\epsilon}_{ii} & \bar{\epsilon}_{ic} \\ \bar{\epsilon}_{ci} & \bar{\epsilon}_{cc} \end{bmatrix} \begin{bmatrix} \bar{e}_i \\ \bar{e}_c \end{bmatrix} + \begin{bmatrix} \bar{i}_i \\ \bar{i}_c \end{bmatrix} \quad (5)$$

where the subscripts  $i$  and  $c$  indicate the internal edges and the edge connected to the lumped element respectively. The first  $i^{\text{th}}$  lines are rewritten as :

$$\partial_t \bar{e}_i = \bar{\epsilon}_{ii}^{-1} \cdot \left[ \left( \bar{C}^t \bar{\mu}^{-1} \bar{b} \right)_i - \bar{i}_i - \partial_t \bar{\epsilon}_{ic} \cdot \bar{e}_c \right] \quad (6)$$

The  $c^{\text{th}}$  line, combined with (6) to remove  $\bar{e}_i$ , gives :

$$\left( \bar{C}^t \bar{\mu}^{-1} \bar{b} \right)_c - \bar{\epsilon}_{ci} \cdot \bar{\epsilon}_{ii}^{-1} \cdot \left[ \left( \bar{C}^t \bar{\mu}^{-1} \bar{b} \right)_i - \bar{i}_i \right] = \partial_t \left[ -\bar{\epsilon}_{ci} \cdot \bar{\epsilon}_{ii}^{-1} \cdot \bar{\epsilon}_{ic} + \bar{\epsilon}_{cc} \right] \bar{e}_c + i_c \quad (7)$$

The equation (7) can be put in the following simple form :

$$I_d = \partial_t C_{\text{int}} \cdot e_c + f(u_c), \quad \text{where } u_c = e_c. \quad (8)$$

For simple lumped elements, the differential equation (8) is solved to find  $e_c$ . For instance, a capacitor  $C$  and a resistor  $R$  are respectively characterized by :

$$i_c = C \cdot \partial_t e_c \quad (9a)$$

$$i_c = e_c / R \quad (9b)$$

Then, the solution  $e_c$  is injected into (6) to build the vector  $\bar{e}_i$ .

But this method is restricted to simple linear or non-linear lumped elements, for which (8) can be integrated analytically. For this purpose, equation (8) is represented by the equivalent circuit, given in Fig. 1, that can be solved by SPICE software. In this equivalent circuit,  $C_{\text{int}}$  is the intrinsic capacitance of the edge connected to the lumped element,  $I_d$  is the conduction current associated with this edge,  $e_c$  and  $i_c = f(e_c)$  are respectively the voltage and the current of the lumped element.

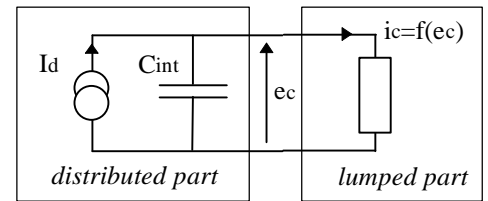


Figure 1 : SPICE equivalent circuit.

This technique presents several advantages. It allows the transient and large frequency band computations and the inclusion of any kind of linear/non linear, passive/active devices without any extra software developments. Moreover, it is well suitable for the investigation of any complex shaped structures, using flexible space discretization.

## RESULTS

To validate our approach, passive lumped loads, placed at the end of a long  $50\ \Omega$  microstrip line (Fig. 2) subjected to a gaussian impulse excitation, are simulated.

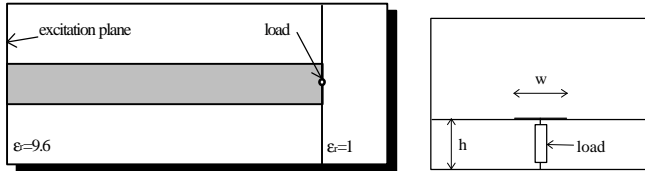


Figure 2 :  $50\ \Omega$  microstrip line loaded by passive lumped element ( $w=h=2\text{ mm}$ ).

Firstly, the FETD-SPICE computed voltage response vs. time across the load is compared to a direct FETD computed response, as explained above. The results, shown for an  $1\text{ nF}$  capacitor and a  $1000\text{ G}\Omega$  resistor respectively in Fig. 3 and Fig. 4, prove the strict equivalence of both approaches. It is noted that the response obtained for the resistor is the same as that obtained at the end of an open-circuited line.

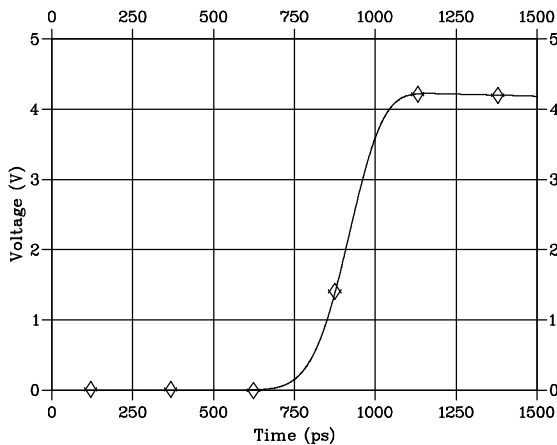


Figure 3 : Voltage response across a  $1\text{ nF}$  capacitor.

continuous line : FETD-SPICE  
continuous line with markers : FETD

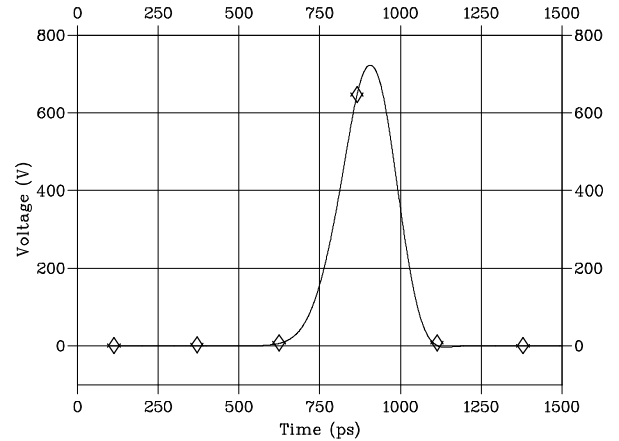


Figure 4 : Voltage response across a  $1000\text{ G}\Omega$  resistor.  
continuous line : FETD-SPICE  
continuous line with markers : FETD

Secondly, the reflected signal response under the line is studied as a function of the location of the lumped load in the distributed structure mesh, using the FETD-SPICE simulation. In the case of the microstrip line loaded with an  $1\text{ nF}$  capacitor, two configurations are considered. The first one corresponds to connecting the capacitor to one edge (Fig. 5) and the second one corresponds to connecting it between two edges (Fig. 6).

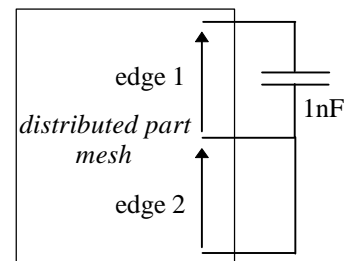


Figure 5 : Capacitor connected to one edge.

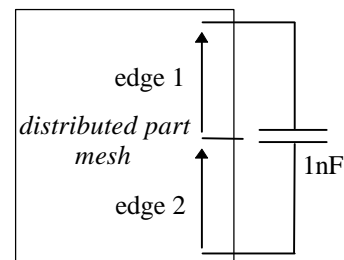


Figure 6 : Capacitor connected to two edges.

The results, given in Fig. 7, show that the assumption of lumped load is satisfactory, since there is no difference between both simulations.

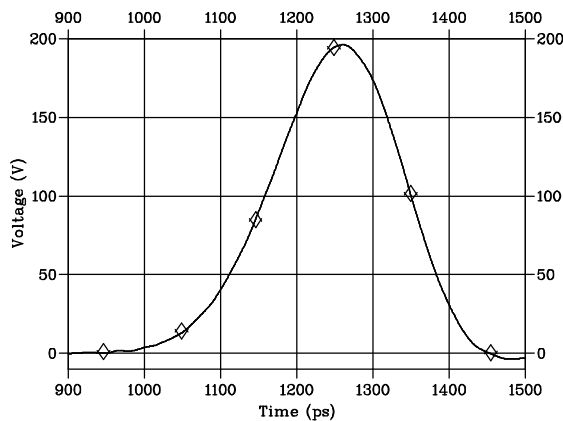


Figure 7 : Reflected signal with a 1 nF capacitor.  
continuous line : load on one cell  
continuous line with markers : load on two cells

Finally, a Schottky-barrier diode ( $I_s=1\text{pA}$ ,  $C_{j0}=0.2\text{pF}$ ,  $V_j=0.7\text{V}$ ), embedded at the end of the line, is simulated without any particular treatment. Only the SPICE input file has to be modified. The expected voltage response across the diode is given in Fig. 8.

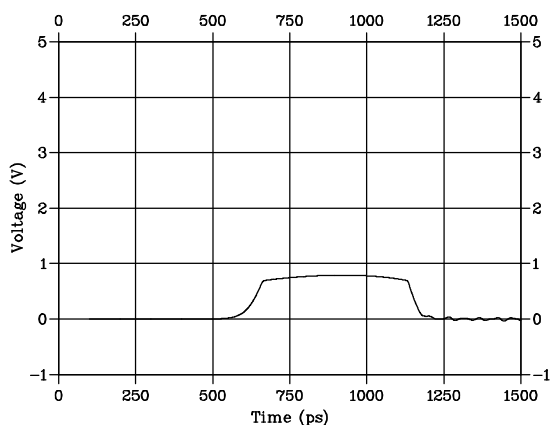


Figure 8 : Voltage response across a Schottky diode with FETD-SPICE simulation.

## CONCLUSION

A new time domain global simulation tool based on a rigorous EM simulator and a powerful circuit software has been presented. This simulator allows precise - transient or large frequency band - characterizations of complex microwave devices, taking into account coupling effects between distributed and lumped parts. It is particularly suitable for the investigation of any non linear circuit.

## REFERENCES

- [1] M-F. Wong, O. Picon, V. Fouad Hanna, "Whitney forms based finite element method in the time domain to solve Maxwell equations", IEEE Trans. on Magnetics, May 1995, vol. 31, n°3, pp. 1618-1621.
- [2] K. Guillouard, M-F. Wong, V. Fouad Hanna, "A new global finite element analysis of microwave circuits including lumped elements", IEEE MTT-S Digest, 1996, pp. 355-358.
- [3] Y. Bégassat, L-A. Ramboz, M-F. Wong, V. Fouad Hanna, J. Citerne, "Modeling of an X-band planar balanced mixer with a mixed SPICE-FDTD program", Microwave and Optical Technology letters, October 1996, vol. 13, n°2, pp. 99-102.
- [4] P. Ciampolini, P. Mezzanotte, L. Roselli, D. Sereni, R. Sorrentino, P. Torti, "Simulation of HF circuits with FDTD technique including non-ideal lumped elements", IEEE MTT-S Digest, 1995, pp. 361-364.
- [5] B. Toland, B. Houshmand, T. Itoh, "Modeling of nonlinear active regions with the FDTD method", IEEE Microwave and Guided Wave Letters, September 1993, vol. 3, n°9, pp. 333-335.
- [6] W. Sui, D. A. Christensen, C. H. Durney, "Extending the two-dimensional FDTD method to hybrid electromagnetic systems with active and passive lumped elements", IEEE Trans. on MTT, April 1992, vol. 40, n°4, pp. 724-730.